Schwartz 8.1 (a)

$k_\mu \epsilon_{\mu\nu} = 0$ removes 1 DOF for each ν index, so removes a total of 4.

$\epsilon_{\mu\nu} = \epsilon_{\nu\mu}$ removes transposing $\epsilon_{\mu\nu}$, so 6 DOF

They in total removes 10 DOF.

Schwartz 8.6(b) We pick $k_\mu = (m, 0, 0, 0)$

then $\varepsilon_{\mu\nu} k_\nu = 0 \Rightarrow \begin{pmatrix} & & & \\ & \varepsilon_{\mu\nu} & & \\ & & & \\ & & & \end{pmatrix} \begin{pmatrix} m \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$,

$$\varepsilon_{\mu\nu} = \begin{pmatrix} 0 & ? & ? & ? \\ 0 & ? & ? & ? \\ 0 & ? & ? & ? \\ 0 & ? & ? & ? \end{pmatrix}$$

by symmetry of $\varepsilon_{\mu\nu}$ ($\varepsilon_{\mu\nu} = \varepsilon_{\nu\mu}$),

$$\varepsilon_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & ? & ? & ? \\ 0 & ? & ? & ? \\ 0 & ? & ? & ? \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & a & b & c \\ 0 & b & d & e \\ 0 & c & e & f \end{pmatrix}$$

$\varepsilon_{ij} = \begin{pmatrix} a & b & c \\ b & d & e \\ c & e & f \end{pmatrix}$ is the degree of freedom we have

Impose orthonormality condition:

$$\varepsilon_{\mu\nu}\varepsilon_{\mu\nu} = \varepsilon_{00}\varepsilon_{00} - \varepsilon_{01}\varepsilon_{01} - \varepsilon_{02}\varepsilon_{02} - \varepsilon_{03}\varepsilon_{03}$$
$$- \varepsilon_{10}\varepsilon_{10} + \varepsilon_{11}\varepsilon_{11} + \varepsilon_{12}\varepsilon_{12} + \varepsilon_{13}\varepsilon_{13}$$
$$- - - \qquad - .$$
$$- - - \ \varepsilon_{32}\varepsilon_{32} + \varepsilon_{33}\varepsilon_{33}$$

$$= a^2 + d^2 + f^2 + 2b^2 + 2c^2 + 2e^2$$

The solutions are

$$\varepsilon_{ij}^{1} = \begin{pmatrix} 1 & & \\ & 0 & \\ & & 0 \end{pmatrix}, \quad \varepsilon_{ij}^{2} = \begin{pmatrix} 0 & & \\ & 1 & \\ & & 0 \end{pmatrix}, \quad \varepsilon_{ij}^{3} = \begin{pmatrix} 0 & & \\ & 0 & \\ & & 1 \end{pmatrix},$$

$$\varepsilon_{ij}^{4} = \begin{pmatrix} 0 & 1/\sqrt{2} & \\ 1/\sqrt{2} & 0 & \\ & & 0 \end{pmatrix}, \quad \varepsilon_{ij}^{5} = \begin{pmatrix} 0 & & 1/\sqrt{2} \\ & 0 & \\ 1/\sqrt{2} & & 0 \end{pmatrix}, \quad \varepsilon_{ij}^{6} = \begin{pmatrix} 0 & & \\ & 0 & 1/\sqrt{2} \\ & 1/\sqrt{2} & 0 \end{pmatrix}$$